SYNTHETIC DIVISION

General steps for dividing synthetically:

1. Arrange the coefficients of the polynomial to be divided in order of descending powers of the variable, supplying zero as the coefficient of each missing power.

2. Use the constant term of the divisor with its sign changed. **Note:** The coefficient and the power of the variable term of the divisor must be 1. (Example: $x + 2$ or $x - 6$)

3. Bring down the coefficient of the largest power of x, multiply it by the divisor, place the product beneath the coefficient of the second largest term, and add. Multiply the sum by the divisor, and place the product beneath the next largest power of x. Continue this procedure until there is a product added to the constant term.

4. The last number in the third row is the remainder, and the other numbers, reading from left to right, are the coefficients of the quotient, which is of one degree less than the given polynomial.

Example:

$$\begin{array}{r|l} & \quad\quad 2x + 3 \\ \hline x+2 & 2x^2 + 7x + 10 \\ & \underline{2x^2 + 4x} \\ & \quad\quad 3x + 10 \\ & \quad\quad \underline{3x + 6} \\ & \quad\quad\quad\quad 4 \end{array}$$

$$\begin{array}{r|rrr} -2 & 2 & 7 & 10 \\ & & -4 & -6 \\ \hline & 2 & 3 & 4 \end{array}$$

Answer: $2x + 3 + \frac{4}{x+2}$

Following the schematic: The constant term of the divisor, with its sign changed, is placed outside the dividend. The coefficients of the dividend are placed in descending order according to the power of the variable. Bring down the 2, multiply it by the divisor(-2), place the -4 below the 7, and add. Multiply the 3 by the -2, place the -6 below the 10, and add. The remainder is 4.

Use synthetic division to find the quotient and any remainders.

1. $(x^3 - 5x^2 + 2x + 8) \div (x - 1)$

2. $(x^4 - x^3 + x - 1) \div (x - 1)$

3. $(3x^4 - 2x^2 - 7x - 2) \div (x - 2)$

THE BINOMIAL EXPANSION

This handout shows how to write and apply the formula for the expansion of expressions of the form $(x+y)^n$ where n is any positive integer. In order to write the formula, we must generalize the information in the following chart:

$$\begin{aligned}
(x+y)^1 &= x+y\\
(x+y)^2 &= x^2+2xy+y^2\\
(x+y)^3 &= x^3+3x^2y+3xy^2+y^3\\
(x+y)^4 &= x^4+4x^3y+6x^2y^2+4xy^3+y^4\\
(x+y)^5 &= x^5+5x^4y+10x^3y^2+10x^2y^3+5xy^4+y^5
\end{aligned}$$

Note: The polynomials to the right have been found by expanding the binomials on the left, we just haven't shown the work.

There are a number of similarities to notice among the polynomials on the right. Here is a list of them:

1. In each polynomial, the sequence of exponents on the variable x decreases to zero from the exponent on the binomial on the left. (The exponent 0 is not shown, since $x^0 = 1$).
2. In each polynomial, the exponents on the variable y increase from 0 to the exponent on the binomial on the left. (Since $y^0 = 1$, it is not shown in the first term).
3. The sum of the exponents on the variables in any single term is equal to the exponent on the binomial to the left.

The pattern in the coefficients of the polynomials in the right can best be seen by writing the right side again without the variables. It looks like this:

$$\begin{array}{c}
1 \quad 1\\
1 \quad 2 \quad 1\\
1 \quad 3 \quad 3 \quad 1\\
1 \quad 4 \quad 6 \quad 4 \quad 1\\
1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1
\end{array}$$

This triangular-shaped array of coefficients is called ***Pascal's Triangle***. Each entry in the triangular array is obtained by adding the two numbers above it. Each row begins and ends with the same number, 1. If we were to continue Pascal's Triangle, the next two rows would be:

$$\begin{array}{c}
1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1\\
1 \quad 7 \quad 21 \quad 35 \quad 35 \quad 21 \quad 7 \quad 1
\end{array}$$

Pascal's Triangle can be used to find coefficients for the expansion of$(x+y)^n$. The coefficients for the terms in the expansion of$(x+y)^n$ are given in the n^{th} row of Pascal's Triangle. Here are more:

$$\begin{array}{c}
1 \quad 8 \quad 28 \quad 56 \quad 70 \quad 56 \quad 28 \quad 8 \quad 1\\
1 \quad 9 \quad 26 \quad 84 \quad 126 \quad 126 \quad 84 \quad 26 \quad 9 \quad 1\\
1 \quad 10 \quad 45 \quad 120 \quad 210 \quad 252 \quad 210 \quad 120 \quad 45 \quad 10 \quad 1
\end{array}$$