Power Series

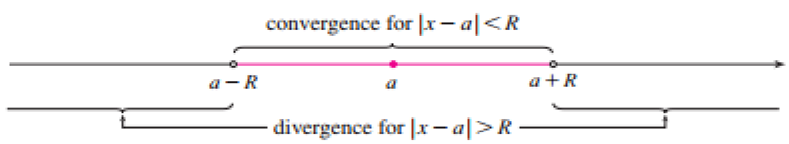
A **power series** is any series of the form:

$$\sum_{n=0}^{\infty} c_n(x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \cdots$$

Three Possible Cases for Convergence

(i) The series converges only when $x = a$.
(ii) The series converges for all x.
(iii) The series converges only within a certain interval of x values, having a radius of convergence R. *The endpoints of the interval must be tested for convergence.*

The **Radius of Convergence** of a power series is a positive number R such that the series converges if $|x-a| < R$ and diverges if $|x-a| > R$.



The series may or may not converge if $|x-a| = R$. In general, either the Ratio Test or the Root Test is used to find the radius of convergence R.

The **Interval of Convergence** of a power series is the interval that consists of all values of x for which the power series converges.

Finding the Interval of Convergence

- Use the Ratio or Root Test to find the radius of convergence, R.
- Take $a - R$ and $a + R$ to find the interval of convergence.
- Check endpoints of the interval if applicable:
  - Plug in endpoints for x.
  - Test for convergence at both values.
  - If the series converges at an endpoint, include the value in the interval of convergence.
  - If the series diverges at an endpoint, do not include the value in the interval of convergence.

Case	Radius of Convergence	Interval of Convergence
The series converges only when $x = a$	0	a
The series converges for all x	∞	$(-\infty, \infty)$
There is a positive number R such that the series converges if $\lvert x-a\rvert < R$ and diverges if $\lvert x-a\rvert > R$	R	$(a-R, a+R)$, $(a-R, a+R]$, $[a-R, a+R)$, OR $[a-R, a+R]$